

Running coupling for Wilson fermions *

Juri Rolf and Ulli Wolff

Humboldt-Universität zu Berlin, Institut für Physik,
Invalidenstrasse 110, 10115 Berlin

July 16, 1999

Abstract

A non perturbative finite size scaling technique is used to study a running coupling in lattice Yang-Mills theory coupled to a bosonic Wilson spinor field in the Schrödinger functional scheme. This corresponds to two negative flavours. The scaling behaviour in this case is compared to quenched results and to QCD with two flavours. The continuum limit is confronted with renormalized perturbation theory.

1 Introduction and motivation

One of the main goals of the ALPHA collaboration is the non perturbative computation of the strong coupling α_S for energy scales ranging from hadronic scales to perturbative high energy scales. To this end a finite volume scale dependent renormalization scheme for QCD has been invented [1, 2]. Its central object is the step scaling function Σ which describes the running of the coupling under a discrete change of the scale. Other ingredients include Schrödinger functional boundary conditions, $O(a)$ improvement and non perturbative renormalization. This method has been used successfully in the quenched approximation (see [3] for a review). First results for full QCD with two flavours have been obtained recently [4]. In figure 1 the results for the step scaling function at the coupling $\bar{g}^2 = 0.9793$ are shown and compared with the quenched approximation and the perturbative continuum limit¹. Here an $O(a)$ improved action was used. The observed cut-off effects are larger than in the quenched approximation. A naive extrapolation of the Monte Carlo data to the continuum limit yields a value which lies 2-3% above the perturbative estimate, although the coupling \bar{g}^2 is expected to be small enough to be in the perturbative regime. Furthermore,

*Talk given at the International Symposium on Lattice Field Theory, June 28–July 3, 1999, in Pisa, Italy.

¹These data were obtained in collaboration with A. Bode, R. Frezzotti, M. Guagnelli, K. Jansen, M. Hasenbusch, J. Heitger, R. Sommer and P. Weisz.

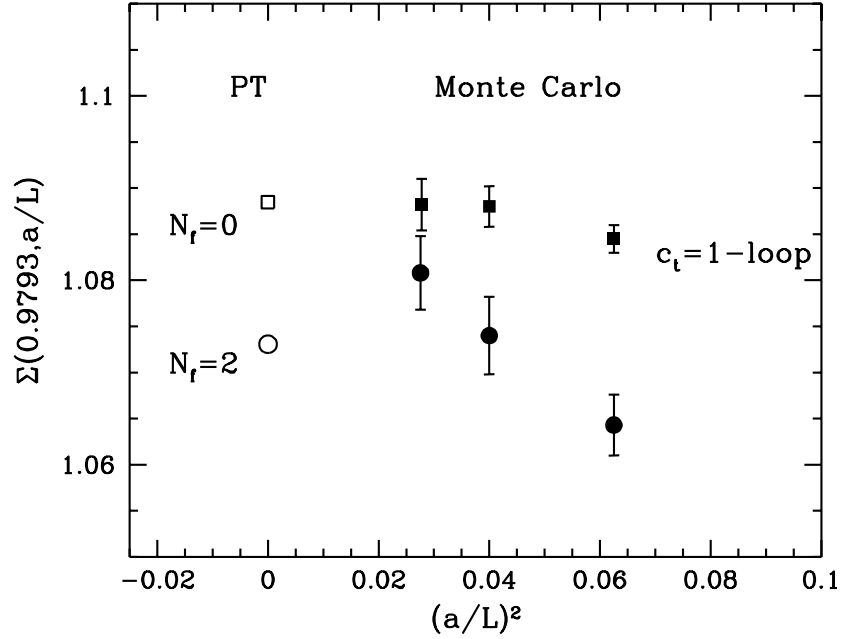


Figure 1: Preliminary results for the step scaling function in QCD with two flavours at $u = 0.9793$ compared with the quenched approximation. Throughout this paper we denote $N_f = 2$ by circles, $N_f = 0$ by squares and $N_f = -2$ by triangles.

the cut-off effects in the quenched and in the full theory computed in lattice perturbation theory to two loops are of the same size [5]. These observations can be interpreted as follows:

- statistical fluctuation
- true 3% deviation from renormalized perturbation theory
- lattice artifacts too large for continuum extrapolation

The investigation of these possibilities in full QCD would consume a substantial amount of computer time. Therefore we use Yang-Mills theory coupled to a bosonic Wilson spinor field, which corresponds to setting $N_f = -2$ in the QCD partition function, as a toy model to study the extrapolation to the continuum limit for a system different from pure gauge theory. This model has been called the bermion model in the literature [6, 7].

2 The bermion model in the Schrödinger functional setup

Let the space time be a hypercubic Euclidean lattice with lattice spacing a and volume $L^3 \times T$. In the following we set $T = L$. The $SU(3)$ gauge field $U(x, \mu)$ is defined on the links while the bermion field $\phi(x)$ which is a bosonic spinor field with color and Dirac indices is defined on the sites of the lattice. In the space directions we impose periodic boundary conditions while in the time direction we use Dirichlet boundary conditions. The boundary gauge fields can be chosen such that a constant color electric background field is enforced on the system which can be varied by a dimensionless parameter η [1].

As explained above our goal is to continue the exponent of the fermion determinant in the QCD partition function to the negative value $N_f = -2$. This can be achieved by integrating the bosonic field ϕ with the Gaussian action

$$S_B = \sum_x |M\phi(x)|^2, \text{ where} \quad (1)$$

$$\frac{1}{2\kappa} M\phi(x) = (\mathcal{D} + m_0)\phi(x). \quad (2)$$

\mathcal{D} is the Wilson Dirac operator with hopping parameter $\kappa = (8 + 2m_0)^{-1}$. For the gauge fields we employ the action

$$S_G = \frac{1}{g_0^2} \sum_p w(p) \text{tr}(1 - U(p)). \quad (3)$$

The weights $w(p)$ are defined to be one for plaquettes p in the interior and they equal c_t for time like plaquettes attached to the boundary. The choice $c_t = 1$ corresponds to the standard Wilson action. However, c_t can be tuned in order to reduce lattice artifacts. For the bermion case we have in this work always chosen $c_t = 1$.

Now the Schrödinger functional is defined as the partition function in the above setup:

$$Z = \int \mathcal{D}U \mathcal{D}\phi \mathcal{D}\phi^\dagger e^{-S_G - S_B} \quad (4)$$

$$= \int \mathcal{D}U e^{-S_G(U)} \det(M^\dagger M)^{N_f/2}, \quad (5)$$

with $N_f = -2$. The effective action

$$\Gamma = -\log Z \quad (6)$$

with the perturbative expansion

$$\Gamma = g_0^{-2} \Gamma_0 + \Gamma_1 + g_0^2 \Gamma_2 + \dots \quad (7)$$

is renormalizable with no extra counterterms up to an additive divergent constant. That means that the derivative $\Gamma' = \frac{\partial \Gamma}{\partial \eta}$ is a renormalized quantity and

$$\bar{g}^2(L) = \Gamma'_0 / \Gamma'|_{\eta=0} \quad (8)$$

defines a renormalized coupling which depends only on L and the fermion mass m . Note that this coupling can be computed efficiently as the expectation value $\frac{\partial \Gamma}{\partial \eta} = \left\langle \frac{\partial S}{\partial \eta} \right\rangle$. The mass m is defined via the PCAC relation [8]. Here we use the fermionic boundary states of the Schrödinger functional² to transform this operator relation to an identity (up to $O(a)$) for fermionic correlation functions which can be computed on the lattice. This gives a time dependent mass $m(x_0)$. The mass m is then defined by

$$m = \begin{cases} m(\frac{T}{2}) & T \text{ even,} \\ \frac{1}{2} (m(\frac{T-1}{2}) + m(\frac{T+1}{2})) & T \text{ odd.} \end{cases} \quad (9)$$

To define the step scaling function $\sigma(s, u)$ let $u = \bar{g}^2(L)$ and $m(L) = 0$. Then we change the length scale by a factor s and compute the new coupling $u' = \bar{g}^2(sL)$. The lattice step scaling function Σ at the resolution L/a is defined as

$$\Sigma(s, u, a/L) = \bar{g}^2(sL) \Big|_{\bar{g}^2(L)=u, m(L)=0}. \quad (10)$$

Note that the two conditions fix the two bare parameters g_0 and κ . The continuum limit $\sigma(s, u)$ can be found by an extrapolation in a/L . That means the computational strategy is as follows:

1. Choose a lattice with L/a points in each direction.
2. Tune the bare parameters g_0 and κ such that the renormalized coupling $\bar{g}^2(L)$ has the value u and $m(L) = 0$.
3. At the same value of g_0 and κ simulate a lattice with twice the linear size and compute $u' = \bar{g}^2(2L)$. This gives $\Sigma(2, u, \frac{a}{L})$.
4. Repeat steps 1.-3. with different resolutions L/a and extrapolate $\frac{a}{L} \rightarrow 0$, which yields $\sigma(2, u)$.

3 Results

We have performed Monte Carlo simulations on APE100/Quadrics parallel computers with SIMD architecture and single precision arithmetics. The size of the machines ranged from 8 to 256 nodes. The gauge fields and the fermion fields have been generated by hybrid overrelaxation including microcanonical reflection steps. While we measured the gauge observables after each update of the

²Fermionic observables are constructed independently of the number of dynamical flavours N_f .

fields the fermionic correlation functions were determined only every 100th to 1000th iteration, since their measurement involves the inversion of the Dirac operator. The statistical errors have been determined by a direct computation of the autocorrelation matrix. The largest run has been performed for the lattice size $L = 24$ which took about 12 days on the largest machine.

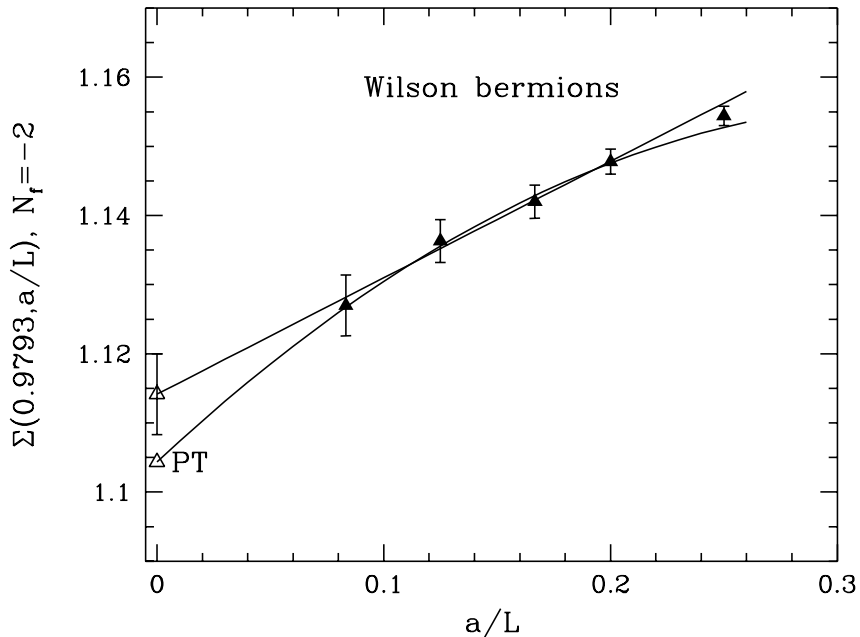


Figure 2: Results for the lattice step scaling function for $N_f = -2$ at $u = 0.9793$ confronted with renormalized perturbation theory.

In figure 2 our results for the fermion lattice step scaling function $\Sigma(2, 0.9793, a/L)$ are shown for resolutions 4, 5, 6, 8 and 12 and compared with renormalized perturbation theory. The lattice artifacts are consistent with $O(a)$ effects. Included is a linear fit to the data which extrapolates to a continuum limit which lies above the perturbative estimate but is compatible with it. Furthermore a linear plus quadratic fit with the constraint that the continuum limit is given by the perturbative estimate is shown. All the data points are compatible with this fit. Note that $L/a = 4$ has been ignored in these fits.

In figure 3 we compare these data with results from the quenched approximation with the standard Wilson action and with a perturbatively $O(a)$ improved action. Again the most naive extrapolation of the unimproved data would extrapolate to a value slightly above the perturbative estimate. However, if we use universality, i.e. the agreement of the continuum limit of the two data sets as

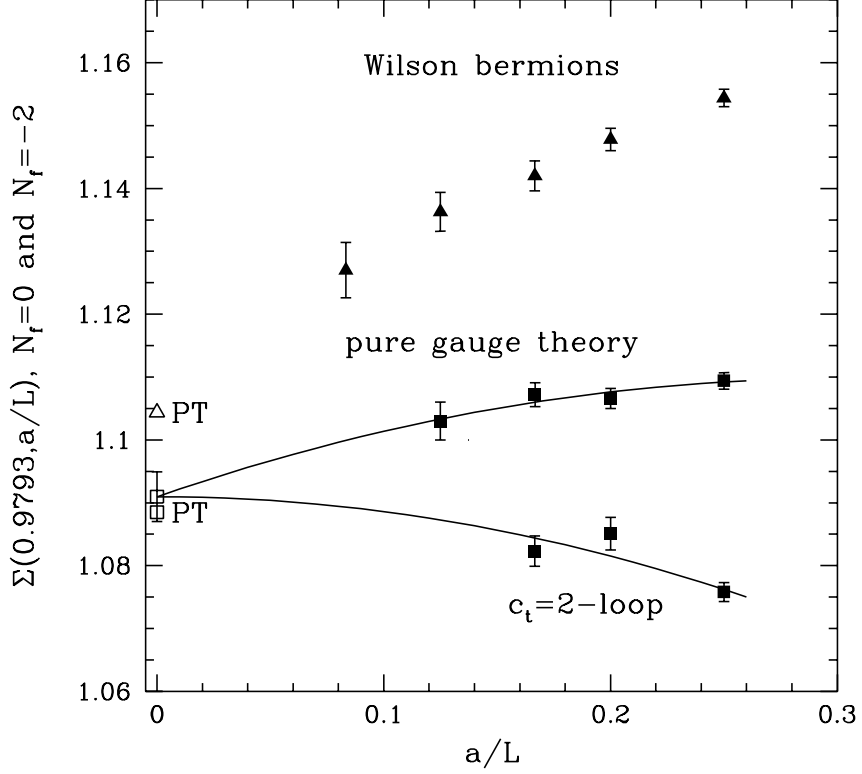


Figure 3: Results for the lattice step scaling functions for $N_f = 0$ in the unimproved and the improved case at $u = 0.9793$ confronted with renormalized perturbation theory.

a constraint, their joint continuum limit is fully compatible with perturbation theory.

Furthermore we observe that in the fermion theory the cut off effects are larger than in the quenched approximation.

Since the extrapolation to the continuum limit in the quenched approximation is much easier in the improved case we plan to study the $O(a)$ improved fermion model. This will also allow us to compare with the fermionic theory on equal footing.

This work is part of the ALPHA collaboration research program. We thank DESY for allocating computer time to this project and the DFG under GK 271 for financial support.

References

- [1] M. Lüscher, R. Narayanan, P. Weisz and U. Wolff, Nucl. Phys. **B384** (1992) 168 hep-lat/9207009.
- [2] M. Lüscher, R. Sommer, P. Weisz and U. Wolff, Nucl. Phys. **B413** (1994) 481 hep-lat/9309005.
- [3] R. Sommer, hep-ph/9711243.
- [4] K. Jansen and R. Sommer, Nucl. Phys. **B530** (1998) 185 hep-lat/9803017.
- [5] A. Bode and U. Wolff, contribution to these proceedings.
- [6] S.J. Anthony, C.H. Llewellyn Smith and J.F. Wheeler, Phys. Lett. **116B** (1982) 287.
- [7] G.M. de Divitiis, R. Frezzotti, M. Guagnelli, M. Masetti and R. Petronzio, Nucl. Phys. **B455** (1995) 274 hep-lat/9507020.
- [8] K. Jansen *et al.*, Phys. Lett. **B372** (1996) 275 hep-lat/9512009.